ADG 2018

12th International Conference on Automated Deduction in Geometry

Nanning, China, September 11-14, 2018
http://adg2018.cc4cm.org/

In Memory of Wen-tsün Wu

Schedule and Abstracts

September 12, morning

9:00-9:20 Opening (Chair: Dongming Wang)

Session 1 (Chair: Hongbo Li)

9:20-10:20 **Plenary Talk**
Wen-Tsun Wu and Mathematics Mechanization
Xiao-Shan Gao

10:20-10:50 Break

10:50-11:50 **Plenary Talk**
Around Dandelin-Gallucci Theorem
Pascal Schreck

12:00 Lunch

September 12, afternoon

Session 2 (Chair: Xiaoyu Chen)

2:00-2:30
Geometric search in TGTP
Yannis Haralambous, Pedro Quaresma

2:30-3:00
Automated Geometer, a web-based discovery tool
Francisco Botana, Zoltán Kovács, and Tomás Recio

3:00-3:30
User interface and operations of GeometryTouch on small and large touchscreens
Wei Su, Chuan Cai and Jinzhao Wu

3:30-4:00 Break

Session 3

4:00-4:30 (Chair: Pascal Schreck)
Exploring algebraic tools for three-dimensional projective geometry
Hongbo Li

4:30-5:00

N-dimension area method using exterior products
V. Pavan

5:00-5:30
Computation of gcd chain over the power of an irreducible polynomial
Xavier Dahan

5:40 Dinner

September 13, morning

Session 4 (Chair: Dingkang Wang)

9:00-10:00 Plenary Talk
Can You Pave the Plane Nicely with Identical Tiles
Chuanming Zong

10:00-10:30 Break

10:30-11:30 Plenary Talk
Interactive Theorem Proving in Geometry: From Foundations to Applications
Jacques Fleuriot

11:40 Lunch

September 13, afternoon

Session 5 (Chair: Jacques Fleuriot)

2:00-2:30
Mining geometry theorems in a regular polygon
Zoltán Kovács

2:30-3:00
On n-sectors of the angles of an arbitrary triangle
Dongming Wang, Bo Huang and Xiaoyu Chen

3:00-3:30
A system for automated deduction in engineering mechanics
Philip Todd

3:30-4:00 Break

Session 6 (Chair: Jing Yang)

4:00-4:30
Towards a mechanisation in Isabelle of Birkhoff’s ruler and protractor geometry
Imogen I. Morris, Jacques D. Fleuriot

4:30-5:00
Mechanising an independent axiom system for Minkowski space-time
Jake Palmer, Jacques D. Fleuriot

5:00-5:30
Verification of natural styled proofs in basic calculus using Maple
Shuai Shi

5:30-6:30 ADG Business Meeting (Chair: Dongming Wang)

7:00 Conference Dinner

September 14, morning

Session 7 (Chair: Dongming Wang)

9:00-10:00 Plenary Talk
Comprehesive Gröbner Systems and Discovering Geometric theorems Mechanically
Dingkang Wang

10:00-10:30 Break

10:30-11:30 Plenary Talk
Hard Combinatorial Problems via SAT
Ilias Kotsireas

11:30-12:00 Closing
Dongming Wang

12:00 Lunch

Abstracts of Plenary talks

Wen-Tsun Wu and Mathematics Mechanization
Xiao-Shan Gao

In this talk, I will give a review of some of the major advances in the field of mathematics mechanization coined by Wen-Tsun Wu. These include the Ritt-Wu characteristic set method for symbolic solution of algebraic, differential, and difference polynomial equation systems; methods for automated proving and discovering geometry theorems; and applications in computer aided geometric design, computer vision, robotics, etc.

Around Dandelin-Gallucci Theorem
Pascal Schreck

This talk is about mechanization of projective incidence geometry. I present some parts of a work which began in 2004 with a proposal of Dominique Michelucci about the so-called hexamys and a study around combinatorial proofs in incidence geometry. Most of this presentation relates to joint works with other members of the Strasbourg team and especially David Braun and Nicolas Magaud. After a brief description of projective incidence geometry, and after recalling the importance of both Desargues and Pappus theorems, I explain how projective incidence geometries correspond to a certain class of matroids. Then, I present Dandelin-Gallucci theorem with several proofs using very different approaches: combinatoric algebra, synthetic geometry, matroids, ...

Can You Pave the Plane Nicely with Identical Tiles
Chuanming Zong

Everybody knows that identical regular triangles or squares can tile the whole plane. Many people know that identical regular hexagons can tile the plane properly as well. In fact, even the bees know and use this fact! Is there any other convex domain which can tile the Euclidean plane? Of course, there is a long list of them! To find the list and to show the completeness of the list is a unique drama in mathematics, which has lasted for more than one century and the completeness of the list has been mistakenly announced not only once! Up to now, the list consists of triangles, quadrilaterals, fifteen types of pentagons, and three types of hexagons. In 2017, Michaël Rao announced a computer proof for the completeness of the list. Meanwhile, Qi Yang and Chuanming Zong made a series of unexpected discoveries in multiple tilings in the

Euclidean plane. For examples, besides parallelograms and centrally symmetric hexagons, there is no other convex domain which can form any two-, three- or four-fold translative tiling in the plane. However, there are two types of octagons and one type of decagons which can form nontrivial five-fold translative tilings. Furthermore, a convex domain can form a five-fold translative tiling of the plane if and only if it can form a five-fold lattice tiling. In this talk we will report these progresses.

Interactive Theorem Proving in Geometry: From Foundations to Applications
Jacques Fleuriot

In this talk, I'll argue that interactive theorem proving is an effective tool for the systematic investigation of geometric problems, ranging from axiomatic foundations to formal verification. This type of mechanization is usually carried out within the settings of proof-assistants such as Isabelle, which provide both a rich language for formalizing non-trivial concepts, e.g. higher-order geometric axioms or even inductive definitions, and an array of powerful automated tools, e.g. first order theorem provers and decision procedures, that can help the user in their quest for a (readable) proof.

I will illustrate the above by discussing some of the achievements from the past twenty years, a number of which were originally presented at Automated Deduction in Geometry. Time permitting, I'll also talk about some potential mechanization challenges and avenues for collaborative proof efforts in geometry.

Comprehesive Gröbner Systems and Discovering Geometric theorems Mechanically
Dingkang Wang

For many geometric theorems, the hypotheses can be represented by a system of parametric polynomial equations and the conclusion can be represented by a parametric polynomial equation. An important problem concerning proving geometric theorems is to determine whether a geometric statement is valid under a specialization of parameters. Comprehensive Gröbner system is an important tool to solve the problem related to parametric polynomial system. We will review the progresses in comprehensive Gröbner systems and then use it to discover geometric theorems mechanically, i.e., we can find out complementary conditions on the parameters such that the geometric statement becomes true or true on components.

Hard Combinatorial Problems via SAT
Ilias Kotsireas

The area of boolean satisfiability and SAT solvers has seen dramatic advances in the past two decades. A recent trend in SAT solving is an attempt to combine the strengths of symbolic computation tools with the power of SAT solvers, in order to improve their effectiveness and to build custom-tailored SAT solvers for hard combinatorial problems. We will describe our work in this context, with a focus on some particularly hard combinatorial problems, described via autocorrelation of finite sequences.

Based on joint work with Vijay Ganesh (University of Waterloo) and Curtis Bright (University of Waterloo) in the context of the Horizon 2020 EU project "Satisfiability Checking and Symbolic Computation" (SC^2).

Abstracts of Regular talks

September 12

Geometric search in TGTP
Yannis Haralambous, Pedro Quaresma

In this age of information the importance of retrieve the knowledge from the many sources of information is paramount. In Geometry, apart from textual approaches, common to other areas of mathematics, there is also the need for a geometric search approach, i.e. semantic searching in a corpus of geometric constructions. The Web-based repository of geometric problems Thousands of Geometric problems for geometric Theorem Provers (TGTP) has, from the start, some text search mechanisms. Since version 2.0 an implementation of the geometric search mechanism is integrated in it. Using a dynamic geometry system it is possible to build a geometric construction and then semantically search in the corpus for geometric constructions that are super-sets of the former, with regard to geometric properties. It should be noted that this is a semantic check, the selected constructions may not look like the query construction, but they will possess similar sets of geometric properties.

Automated Geometer, a web-based discovery tool
Francisco Botana, Zoltán Kovács, and Tomás Recio

The goal of this communication to ADG 2018 is to report on-going work by the authors towards the automated and systematically finding of properties on a given geometric construction. Our Automated Geometer is being implemented on top of GeoGebra, of a dynamic geometry system with millions of users at high schools and universities. It exploits GeoGebra recently added functionalities regarding automated reasoning tools, providing rigorous, symbolic-driven, proofs of geometric facts. In the talk we will illustrate and describe some basic facts about the system we are developing.

User interface and operations of GeometryTouch on small and large touchscreens
Wei Su, Chuan Cai and Jinzhao Wu

GeometryTouch is Web-based geometry education system, which can run on Chrome or Safari browsers of touch devices. The paper introduces the user interface design principle and operation method of GeometryTouch on small and large touchscreens. Firstly, we conduct a brief analysis of the differences between mobile small devices such as 5-inch smartphones and very large touchscreen de-vices such as 70-inch smart whiteboards in the paper. Secondly, the paper gives some useful strategies for the user interface and operation design of GeometryTouch on both size devices.

Exploring algebraic tools for three-dimensional projective geometry
Hongbo Li

The algebras tools for three-dimensional projective geometry include not only matrix algebra, but also Grassmann algebra, Clifford algebra, etc. This talk first presents our recent work on symbolically computing all pre-images of the exponential map from sl(4) to SL(4) for any input matrix in SL(4), then presents our recent result on minimal spinor generation of all three-dimensional projective transformations in the Plücker model.

N-dimension area method using exterior products
V. Pavan

In this document we present a generalization of the axioms of the Aera Method in any dimension using the interpretation of the exterior (Grassmann) product between vectors.

Computation of gcd chain over the power of an irreducible polynomial
Xavier Dahan

A notion of gcd chain has been introduced by the author at ISSAC 2017 for two univariate monic polynomials with coefficients in a ring $R = k[x_1, . . . , x_n]/\langle T \rangle$ where T is a primary triangular set of dimension zero. A complete algorithm to compute such a gcd chain remains challenging. This work treats the case of a triangular set T = (T1(x)) in one variable, namely a power of an irreducible polynomial. This seemingly easy case is nonetheless essential for considering the general case.

September 13

Mining geometry theorems in a regular polygon
Zoltán Kovács

We demonstrate a systematic, automated way of discovery of geometry theorems on regular polygons.

On n-sectors of the angles of an arbitrary triangle
Dongming Wang, Bo Huang and Xiaoyu Chen

Morley's theorem shows that the three points, each of which is the intersection of the two internal trisectors that are the closest to the same side of an arbitrary triangle Δ, form an equilateral triangle. This beautiful theorem was proved mechanically by Wu in 1984 in the most general form: the neighbouring trisectors of the three angles of Δ intersect to form 27 triangles in all, of which 18 are equilateral triangles, called Morley triangles. A natural question is: does there exist any equilateral triangle, other than Morley triangles, which is formed by some intersection points of the neighbouring angular n-sectors of Δ for $n \geq 3$? In this paper, we approach this question using specialized techniques with interactive, semi-automatic algebraic computations and prove that for $n = 4$ and 5 the three points, each of which is the intersection of the two internal (or two external) angular n-sectors closest to the same side of Δ, form an equilateral triangle if and only if Δ is equilateral. The computational approach we present can also be applied to other cases for specific n. How to establish the non-existence of equilateral triangles formed by the intersection points of angular n-sectors for general n remains to be an interesting question.

A system for automated deduction in engineering mechanics
Philip Todd

A system is presented for automated formula discovery in engineering mechanics built on a Lagrangian formulation where geometric constraints are treated as physical constraints. While the architecture of the system is described, the main focus of this paper is to highlight the interplay between architecture and user interface in generating formulas for mechanical problems. With this in mind, a number of examples are presented in the kinematics, statics and dynamics of simple mechanisms.

Towards a mechanisation in Isabelle of Birkhoff's ruler and protractor geometry
Imogen I. Morris and Jacques D. Fleuriot

We report a work-in-progress formalisation of Birkhoff's axioms for metric Euclidean geometry in the interactive theorem prover Isabelle. Because his axioms are strong, it gives us a head start in proving high-level theorems, without needing to build up as far from the foundations as in the axiom systems given by Euclid, Hilbert or Tarski. Our formalisation begins with the axioms on line measure, includes theorems on angle measure and finally shows that the measure of a straight angle is π. Birkhoff's presentation is sometimes hard to follow so we use instead a more precise rewriting of his axioms given by Brossard in 'Birkhoff's Axioms for Space Geometry'.

Mechanising an independent axiom system for Minkowski space-time
Jake Palmer and Jacques D. Fleuriot

We describe a work in progress that takes the first steps in implementing and investigating an axiomatisation of Minkowski spacetime whose primitive undefined basis consists of a set of events, a set of paths consisting of events, and a ternary betweenness relation. There are 15 independent axioms in total: 6 so-called axioms of order, 7 axioms of incidence, the axiom of symmetry, and the axiom of continuity. We describe Minkowski space-time and how it relates to special relativity, formalise and correct some of the axioms presented by the original author of the system, develop his proofs, and fill in some gaps. Ultimately the purpose of this work is to try to push and explore the boundaries of automated reasoning in physics. The result is a starting point for a new formal, mechanised foundations for Minkowski space-time in Isabelle/HOL.

Verification of natural styled proofs in basic calculus using Maple
Shuai Shi

In verifying mathematical proofs, current proof assistant systems are generally based on logic approach, and short of symbolic computation power. Maple, on the other hand, is a powerful symbol computation system, but lacks packages for formal mathematical proof verification. In natural styled proofs in basic calculus, both logic reasoning and pure symbolic computing occur, and the latter weighs more. Instead of doing logic reasoning verification in a proof assistant system such as Mizar, while doing symbolic computing verification in Maple, in this talk we introduce our recent work of doing both verifications in Maple, by developing a package of symbolically computing sufficient conditions for the correctness of the inference of a natural styled proof in basic calculus.